

Black hole complementarity: the inside view

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Abstract

Within the framework of black hole complementarity, a proposal is made for an approximate interior effective field theory description. For generic correlators of local operators on generic black hole states, it agrees with the exact exterior description in a region of overlapping validity, up to corrections that are too small to be measured by typical infalling observers.

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I. INTRODUCTION

Black hole complementarity posits that a unitary and local description of physics exists outside a stretched horizon, a timelike surface a short distance from the event horizon of a black hole. The postulates of [1] leave open the question of how to describe the physics inside the horizon but based on the equivalence principle it is reasonable to expect that a freely falling observer experiences nothing out of the ordinary when crossing the horizon of a sufficiently large black hole. If this expectation is indeed borne out, it also seems reasonable that observations made inside a laboratory that enters a black hole in free fall should be described, to within achievable experimental precision, by a more or less conventional effective field theory. It was already observed in [1] that this effective description cannot be a local quantum field theory that is simultaneously valid for distant observers and observers who have entered the black hole in free fall. The problems that arise when one attempts to implement unitary black hole evolution from the point of view of distant observers in the context of a local effective field theory that extends into the black hole interior were stated more sharply in [2], where it was pointed out that observations made on the outgoing Hawking radiation would project the quantum state of the black hole and in effect burn up the inside observer. In fact, no explicit measurements are needed - the effect follows from decoherence due to the local coupling between the Hawking radiation and degrees of freedom far from the black hole. More recently similar conclusions were reached in [3, 4] by considering the entanglement between outgoing Hawking modes at different times during the evaporation. An alternative conclusion is that there is no firewall but that the problem lies with applying local effective field theory across the horizon [2, 5].

In the present work we construct an approximate effective field theory for an observer who passes through the black hole horizon in free fall. The construction follows up on our recent work in [6] where the evolution of a black hole formed in a generic pure state was considered and it was argued that a typical infalling observer would not see any drama on their way towards the stretched horizon. While this is a satisfying conclusion it does not answer the key question of what happens to such an observer in the interior region, which we take to include both the black hole region inside the event horizon and the region between the event horizon and the stretched horizon. In order to address that question we need to have a model for the interior quantum evolution and the answer turns out to depend on

the model. If we, for instance, choose to use a local quantum field theory on a set of time slices that cover the exterior region during much of the black hole lifetime and also extend smoothly into the black hole region, staying away from the strong curvature near the black hole singularity, then we would conclude that either there is no information about the black hole state carried in the Hawking radiation, as was indeed concluded by Hawking [7], or that the equivalence principle is violated, as was concluded by the authors of [3, 4]. Our construction gets around this by patching together effective field theories on either side of the stretched horizon in such a way that a typical infalling observer will not see any drama until near the black hole singularity. The construction only applies to a restricted class of observers and it is restricted to a set of time slices that only cover a relatively short period of time before and after the observer enters the black hole. Our main claim is that, even with these restrictions imposed, the resulting effective field theory can describe observations made by a typical infalling observer to sufficient accuracy to conclude that no drama is encountered until deep inside the black hole.

An alternative approach to describing the interior physics, inspired by the non-locality of string field theory [8], is to look for a non-local formulation of quantum field theory on a continuous background geometry. For recent work along those lines see [9, 10]. Another approach is that of fuzzball complementarity [11, 12] which uses string theory degrees of freedom to build an interior description.

The paper is organized as follows. In section II some pertinent facts about the geometry of a Schwarzschild black hole are reviewed and a suitable set of timeslices identified for describing infalling observers inside the black hole. These timeslices are different from so called nice time slices [8, 13] that are often invoked in the context of black hole evolution. In particular, they avoid the gravitational back-reaction issues raised in [2, 14] but instead each slice terminates at the singularity from a relatively early time onwards. In section III we briefly review the pull-back/push-forward approach of [15, 16] where an interior description is obtained from a late time state containing outgoing Hawking radiation only by pulling back to a state prior to black hole formation using the exact S-matrix and then pushing forward in time using an approximate local effective Hamiltonian defined on time slices that extend into the black hole interior. In section IV the failure of this approach, when formulated over time scales of order the black hole lifetime, is described in terms of the decoherence/localization of the state with respect to interactions with degrees freedom or

measuring devices in the exterior. An estimate is given of the minimal time it takes for the Hawking particles to localize themselves through interactions with degrees of freedom in the environment outside the black hole. We later argue this decoherence process in the exterior theory coincides with the complementary process of the infalling observer being destroyed at the black hole singularity.

Our recipe for constructing the interior theory applicable to a family of freely falling observers who cross the horizon at some particular time, is described in section V. It involves a combination of a pull-back/push-forward procedure and patching onto a local vacuum configuration inside the stretched horizon. An important new feature is that the late time state is no longer pulled back to a time before the black hole forms but only to a time slice when the infalling observer is within a time of order $M \log M$ of entering the black hole. The patching will give rise to a kind of firewall in the effective description but one that is harmless to a typical infalling observer. Some problems that can arise with this construction for finely tuned states arranged by malevolent outside observers are discussed. Finally, a brief summary and discussion is presented in section VI.

II. BLACK HOLE GEOMETRY AND INFALLING OBSERVERS

A black hole of mass M formed in the gravitational collapse of non-rotating neutral matter in $3 + 1$ dimensional asymptotically flat spacetime will settle down to a metastable state in a time of order M as measured by distant observers and then slowly evaporate due to Hawking emission in a time of order M^3 . During the evaporation, on time scales that are short compared to the black hole lifetime, the geometry is well approximated by the static Schwarzschild solution

$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}} dU dV + r^2 d\Omega^2$$

written here in Kruskal coordinates, related to the familiar Schwarzschild coordinates r, t by

$$\begin{aligned} V &= \left(1 - \frac{r}{2M}\right)^{1/2} e^{\frac{r+t}{4M}} \\ U &= \left(1 - \frac{r}{2M}\right)^{1/2} e^{\frac{r-t}{4M}} \end{aligned}$$

inside the horizon and

$$V = \left(\frac{r}{2M} - 1\right)^{1/2} e^{\frac{r+t}{4M}}$$

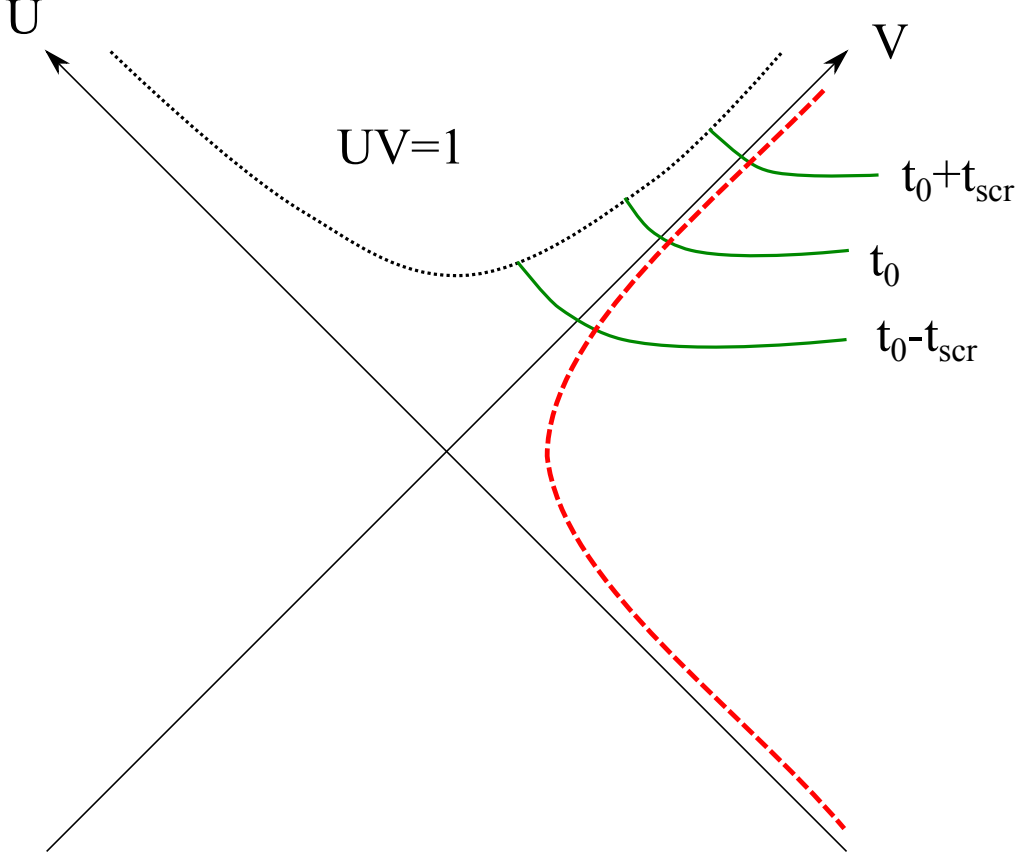


Figure 1: Schematic figure of time slices labelled by Schwarzschild time t outside the stretched horizon and which approach light sheets inside the black hole.

$$U = -\left(\frac{r}{2M} - 1\right)^{1/2} e^{\frac{r-t}{4M}}$$

outside the horizon. In these coordinates, the future event horizon is at $U = 0$ and the curvature singularity on the hyperboloid $UV = 1$. Time translations in Schwarzschild time act as opposite rescalings of U and V .

According to the second postulate of [1], physics outside the so-called stretched horizon is well described by a local effective field theory, which we'll take to have a UV cutoff Λ . The stretched horizon is a timelike surface just outside the event horizon, located where fiducial observers at rest with respect to the black hole would measure a local temperature of order the cutoff scale. In Kruskal coordinates this corresponds to a hyperboloid $UV = -a^2$, where a is a cutoff dependent constant $a \sim (M\Lambda)^{-1/2}$. The effective field theory of the second postulate is only valid outside the stretched horizon and is intended for describing observations made by outside observers. For unitary black hole evolution, it needs to be

supplemented by non-trivial quantum dynamics on the stretched horizon that serves to absorb, thermalize and re-emit the information in infalling matter. This outside effective field theory is not well suited for modeling observations made by infalling observers who enter the black hole, since, in this description, no reference is made to the interior geometry of the black hole. Below, we provide an alternative low-energy effective description, suitable for typical infalling observers, *i.e.* ones who do not carry with them detailed information about the quantum state of the black hole. We refer to the Hamiltonian of the outside effective field theory plus stretched horizon dynamics as the exact Hamiltonian as it generates the exact S-matrix between the initial and final states of the system.

In order to describe infalling observers, we need to introduce a foliation of the spacetime that covers the black hole interior. Following [15], we adopt a set of time-slices, labelled by Schwarzschild time t , that enter the region inside the horizon of the black hole as shown in figure 1. Far outside the black hole the time-slices follow the usual Schwarzschild coordinate system but within a distance of order M from the stretched horizon the slices turn over and join smoothly onto surfaces of constant V inside the stretched horizon.

Consider an observer on the $t = t_0$ time-slice, who enters the black hole in radial free fall at $V = V_0 \gg 1$. At the event horizon the equation for the corresponding radial geodesic simplifies to

$$\frac{dU}{d\tau} = \frac{\alpha}{4MV_0}, \quad \frac{dV}{d\tau} = \frac{eV_0}{4M\alpha},$$

where where $\alpha > 0$ parametrizes the instantaneous velocity and low energy corresponds to $\alpha \sim O(1)$. The worldline is timelike so $dU/d\tau > 0$ everywhere inside the black hole. Assuming the observer stays in free fall for at least a one Planck unit of proper time after passing through the horizon, but allowing for arbitrary timelike motion after that, it follows that the worldline will intersect the singularity at Kruskal retarded time $U > \frac{\alpha}{4MV_0}$. This in turn implies an upper bound on the advanced Kruskal time when the observer runs into the singularity given by $V < \frac{4M}{\alpha}V_0$.

Now consider a signal sent into the black hole at Schwarzschild time $t_0 + t_{scr}$. The advanced Kruskal time at the point, where the signal passes through the event horizon, satisfies $V = e^{\frac{t_{scr}}{4M}}V_0$ and only the region in the forward light-cone of this point on the horizon can be influenced by the signal. Therefore, we see that as long as

$$t_{scr} > 4M \log \frac{4M}{\alpha}$$

the interior observer will have hit the singularity before the signal can have any influence. Now if the observer enters the horizon with a large velocity, this time can be made very long. However, in that case the energy of the observer in the frame of the black hole is at least M_{obs}/α if the rest-mass of the observer is M_{obs} . If we demand the back-reaction on the black hole geometry be negligible, we require

$$M_{obs}/\alpha \ll M$$

and as long as

$$t_{scr} > 8M \log 2M$$

an observer subject to our conditions will always have hit the singularity prior to receiving the signal. We note this time has the same form as the fast scrambling time of [17], explaining our use of the subscript on t_{scr} .

III. PULL-BACK, PUSH-FORWARD

The pull-back/push-forward procedure considered in [15, 16] gives a prescription for computing correlators of local operators on a time slice that extends into the black hole interior starting from data on a late time slice when the black hole has evaporated and the system only contains outgoing Hawking radiation. The first step is to use the S-matrix to pull back to a smooth initial state on an early time slice before the black hole is formed. This state is then evolved forward using the usual low energy effective field theory on the time slices of the previous section. An alternate description, at least for exterior local operators, is provided by evolution with respect to the exact exterior Hamiltonian.

An advantage of this approach is that it can be reformulated when a holographic description of the black hole evaporation is available. The exterior local Hamiltonian density is a local operator that may be reconstructed holographically, as can any other local bulk operator, along the lines of [18] (for recent work on the holographic reconstruction of bulk observables see [19, 20]). Thus the two distinct time evolutions, one with respect to the exact Hamiltonian, and one with respect to the local effective Hamiltonian, are in principle well-defined.

After a Page time, when half the initial entropy of the black hole has emerged in the Hawking radiation, the two approaches disagree when one considers correlators that probe

large numbers of outgoing Hawking particles. In [15], this disagreement was viewed as supportive of the firewall idea. Our construction gets around this problem by restricting the pull-back/push-forward prescription to a finite time interval before and after the infalling observer enters the black hole.

IV. DECOHERENCE AND LOCALIZATION

To better quantify the nature of the disagreement between the two distinct time evolutions it is helpful to consider the decoherence of the quantum state as the outgoing Hawking particles stream out, and potentially interact with measuring apparatus of arbitrarily large size. This idea of decoherence has a long history going back to the work of Mott [21]. He asked the question why do alpha-particle tracks in a cloud chamber appear to be straight lines when they are emitted from a nuclear decay in an s-wave. By considering the interaction of the alpha-particle with the atoms in the cloud chamber, he showed that after essentially a single interaction, a straight line path was picked out, with other contributions to the wavefunction interfering destructively.

In the present situation, we wish to ask how long it will take for interactions of the Hawking particles to localize themselves with respect to some environment. We call this timescale the decoherence time. If left to their own devices, the self-interaction of these Hawking particles is so small that the timescale will easily be longer than the lifetime of the black hole. The question whether an observer propagating will see local quantum mechanics according in their freely falling frame, or something non-local happen as they approach the horizon, boils down to a question of calculating the minimal timescale with which local interactions in the exact theory will lead to a decoherence of the exact state with respect to local interactions in the exterior.

To obtain the minimal timescale that one might achieve in principle, imagine surrounding the black hole with a set of detectors, close to the horizon. Such a set of detectors will behave much like the stretched horizon itself. Specifically, we seek the timescale with which an incoming state hitting the stretched horizon should subsequently decohere due to local interactions of the emitted Hawking particles with the detectors. Since the entanglement is not emitted until after the scrambling time [17], we expect the timescale for decoherence will be bounded below by t_{scr} (with respect to the timeslices of section II).

If we apply this picture to the attempt at reconstructing the black hole interior in section III we immediately see a problem. The Page time is much longer than this decoherence time. Already after t_{scr} the state will effectively decohere due to the local interactions of the exterior Hawking particles with potentially large, localized detectors outside the black hole. Such interactions will appear highly non-local from the viewpoint of the interior effective description. Thus interior observers will not see ordinary quantum evolution with respect to their local Hamiltonian density.

V. PULL-BACK/PUSH FORWARD REVISITED

Let us instead try to introduce the minimal elements needed to build an interior description of the black hole from the point of view of some set of observers close to some pencil of timelike geodesics that cross the horizon. Let such an observer cross the horizon at t_0 , following the discussion of section II where the timeslices of interest are set up. The decoherence arguments indicate that at best we can trust evolution with respect to the local effective Hamiltonian only up to time $t_0 + t_{scr}$.

On the portion of the timeslice outside the stretched horizon at t_0 , we use the initial state inferred from the exact Hamiltonian evolution. In order to fully specify the evolution of such an observer, it is also necessary to specify initial data on the interior portion of the timeslice at t_0 . To do this, we pull-back in time to the timeslice $t_0 - t_{scr}$, using the local effective Hamiltonian. The arguments of section II show that with a reasonable proper distance cutoff, the details of the initial state at $t_0 - t_{scr}$ in the interior are irrelevant once one propagates forward to t_0 for all but a thin layer extending from of order a Planck length inside the global horizon to the stretched horizon.

To specify this remaining initial data at $t_0 - t_{scr}$, we place vacuum initial conditions in this layer. These initial conditions should be determined by the condition that the state be a good approximation to a Hadamard state [22, 23]. It should be noted that such a state leads to a firewall inside the global horizon, as originally suggested in [4]. The condition of a Hadamard state means that the local energy density will be relatively small in the thin layer. Likewise, in the exterior, the arguments of [6] show that the expectation value of the stress tensor seen by a freely falling observer will be very close to the purely thermal result expected in the Hartle-Hawking or Unruh vacua. If one also introduces a Planck scale

smearing in the spatial directions, the computation of [6] shows the correction to the energy density expected, beyond the purely thermal results, will be of order $e^{-S(M)}$ in Planck units, where $S(M)$ is the Bekenstein-Hawking entropy. However as one leaves the layer, moving inward, one encounters modes that are not entangled with their exterior partners, as they would be in the Unruh or Hartle-Hawking vacua, so one expects an energy density there corresponding to an effective temperature of order the stretched horizon cutoff scale.

The beauty of the construction is that the geometry described in section II is such that this interior firewall will hit the singularity before it can interact with our observer entering at t_0 . Taking this initial state at $t_0 - t_{scr}$ and pushing forward to t_0 using the effective local Hamiltonian then leads to a good initial state at t_0 for the infalling observer. In particular, it solves the so-called frozen vacuum problem [24], because the only infalling data that can influence the infalling observer inside the horizon falls in later than $t_0 - t_{scr}$ by section II. Such data will interact and change the state in the interior layer as one evolves forward to t_0 , by which time we will typically have a non-vacuum initial state in the interior.

The recipe described above thus gives a regular time evolution for the interior observer until near the curvature singularity. This local evolution of the interior observer has a non-local interpretation in the exterior stretched horizon theory prior to $t_0 + t_{scr}$, that only comes into conflict with the subsequent emission of Hawking radiation after the time $t_0 + t_{scr}$, as was argued in section IV. By this time, however, the observer has already hit the singularity by the arguments of section II.

It should be noted that the above recipe will only work for typical observers who are not able to measure correlators of a large number of local operators, or resolve differences of order $e^{-S(M)}$ in correlators of small numbers of local operators, since the arguments of [6] are used. The timeslice at t_0 is certainly capable of accommodating large measuring machines, that are not necessarily subject to these restrictions. Correlators of local observables will agree between the low-energy effective description and the exact exterior description in the overlap region outside the stretched horizon between $t_0 - t_{scr}$ and $t_0 + t_{scr}$, unless the local operators are somehow able to probe what is usually nonlocal entanglement between the Hawking particles emitted from the stretched horizon after $t_0 - t_{scr}$ and those emitted earlier. Restrictions on the measurements of such typical observers have also been studied in [12]. The need for a sequence of patches of effective field theories to describe the quantum mechanics of an inside observer was also noted in [25].

For a finely tuned external state, as might be arranged by some large external measuring device, time evolution may lead to an ingoing state entangled with a Hawking particle emerging from the stretched horizon, just as the observer crosses. Such a state will show up as a kind of fireball for the observer. If the argument of typicality of black hole states of [26] is correct, then such fireballs will quickly evolve back to a smooth apparent geometry. The same kind of finely tuned firewall may also be arranged to appear inside the horizon. In this case the entangled pair of modes is inside a future trapped region so both modes will be ingoing. It has been suggested that this kind of fine tuning may require manipulations of the external measurement apparatus that cannot be carried out within the black hole lifetime [27], however the present construction only requires this cannot be done faster than t_{scr} .

VI. CONCLUSIONS

We have presented an approximate effective field theory to model observations made by a typical low-energy observer entering a black hole in free fall at a prescribed time. The effective field theory is allowed to be only approximate because the measurement precision that is available to such an observer is limited both by the finite proper time remaining before hitting the singularity and by the finite size of measuring devices that can be carried into the black hole without significant back-reaction on the geometry [2]. Our construction involves a variant of a pull-back/push-forward procedure that takes into account the minimal decoherence time scale of outgoing Hawking quanta and only operates within a relatively short time interval before and after the infalling observer enters the black hole.

We argue that a typical observer inside a typical black hole will see no quantum drama until they are of order a Planck time from the singularity. On the other hand, an external influence, having acquired precise knowledge of the black hole initial state, is capable of sending in a low energy ingoing component of the state, precisely entangled with some outgoing Hawking particle. While such a process requires extreme fine-tuning, it would cause our recipe for the “inside story” to fail for some particular infalling observer who encounters the resulting firewall. Such a failure is an inevitable consequence of the approximate description of the interior extracted from the exact evolution, and we believe in this case the exception proves the rule.

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- [1] L. Susskind, L. Thorlacius, and J. Uglum, Phys.Rev. **D48**, 3743 (1993), hep-th/9306069.
 - [2] D. A. Lowe and L. Thorlacius, Phys.Rev. **D73**, 104027 (2006), hep-th/0601059.
 - [3] S. L. Braunstein, S. Pirandola, and K. Życzkowski, Phys. Rev. Lett. **110**, 101301 (2013), URL <http://link.aps.org/doi/10.1103/PhysRevLett.110.101301>.
 - [4] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, JHEP **1302**, 062 (2013), 1207.3123.
 - [5] K. Larjo, D. A. Lowe, and L. Thorlacius, Phys. Rev. D **87**, 104018 (2013), URL <http://link.aps.org/doi/10.1103/PhysRevD.87.104018>.
 - [6] D. A. Lowe and L. Thorlacius, Phys.Rev. **D88**, 044012 (2013), 1305.7459.
 - [7] S. Hawking, Phys.Rev. **D14**, 2460 (1976).
 - [8] D. A. Lowe, J. Polchinski, L. Susskind, L. Thorlacius, and J. Uglum, Phys.Rev. **D52**, 6997 (1995), hep-th/9506138.
 - [9] S. B. Giddings, Phys.Rev. **D88**, 024018 (2013), 1302.2613.
 - [10] S. B. Giddings and Y. Shi (2013), 1310.5700.
 - [11] S. D. Mathur and D. Turton (2012), 1208.2005.
 - [12] S. D. Mathur and D. Turton (2013), 1306.5488.
 - [13] R. Wald, (unpublished) (1993).
 - [14] S. B. Giddings and M. Lippert, Phys.Rev. **D69**, 124019 (2004), hep-th/0402073.
 - [15] L. Susskind (2012), 1210.2098.
 - [16] B. Freivogel and L. Susskind, Phys.Rev. **D70**, 126007 (2004), hep-th/0408133.
 - [17] P. Hayden and J. Preskill, JHEP **0709**, 120 (2007), 0708.4025.
 - [18] D. Kabat, G. Lifschytz, and D. A. Lowe, Phys.Rev. **D83**, 106009 (2011), 1102.2910.
 - [19] K. Papadodimas and S. Raju, Phys.Rev.Lett. **112**, 051301 (2014), 1310.6334.
 - [20] K. Papadodimas and S. Raju (2013), 1310.6335.

- [21] N. F. Mott, Proc. R. Soc. Lond. **A126**, 79 (1929).
- [22] B. S. Kay and R. M. Wald, Phys.Rept. **207**, 49 (1991).
- [23] M. Radzikowski, Commun.Math.Phys. **179**, 529 (1996).
- [24] R. Bousso, Phys.Rev.Lett. **112**, 041102 (2014), 1308.3697.
- [25] S. D. Mathur (2013), 1108.0302.
- [26] S. G. Avery and D. A. Lowe (2013), 1310.7999.
- [27] D. Harlow and P. Hayden, JHEP **1306**, 085 (2013), 1301.4504.